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CHARACTERISTICS OF A VISCOPLASTIC MATERIAL IN THE COUETTE FLOW

S. L. Gavrilenko,[1] S. V. Shil'ko,[1] and R. A. Vasin[2]

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A model governing a steady flow of a viscoplastic material between coaxial cylinders is proposed. Nonlinear velocity sensitivity typical of superplastic materials is taken into account. An algorithm of calculating the characteristics of the material is developed. The algorithm is based on the experimental data on moments and angular velocities of the rotating coaxial cylinders. The stability of the algorithm to errors in the initial data is estimated.

Introduction. In some technological processes of plastic metal working, for example, in producing articles by deep drawing, a developed plastic flow occurs. A viscoplastic flow is also observed in producing polymer alloys by mechanical activation in mixers and extruders; a similar flow occurs in a grease layer between machine parts or in a paint layer in printers. Viscous flow is typical of the most technological processes based on the superplasticity phenomenon [1, 2].

In mathematical simulation and optimization of the aforementioned technological processes, it is necessary to know viscoplastic characteristics of the material. To this end, experiments are performed to study the shear state, which occurs, for example, under conditions of the material flow in the clearance between rotating coaxial cylinders (Couette flow) [3, 4]. In these experiments, the angular velocities and moments of rotation of the cylinders are recorded, and the material constants are determined by processing experimental data.

In calculations, a viscoplastic model with nonlinear viscosity and zero yield point is usually used. However, taking into account the critical stress is of principal importance. It is, therefore, necessary to develop a method of determining the characteristics of a viscoplastic material with allowance for the critical stress and nonlinear velocity sensitivity. One of these methods is considered in the present paper.

Formulation of the Problem. We consider a steady flow of a viscoplastic material in the clearance between infinetely long rigid coaxial cylinders of radii a and b ($b > a$), which rotate with angular velocities ω_a and ω_b about the common axis under the action of the moments M_a and M_b, respectively (Fig. 1).

Let the circumferential velocity of the flow be a continuous and monotonic function of the radius in the entire region occupied by the material and adhesion conditions be specified on the boundaries. Leonova [5] determined the conditions under which stagnation zones are absent for the motion of a viscoplastic medium between coaxial cylinders. The values of parameters used in the numerical experiments considered below satisfy these conditions.

We introduce a cylindrical system of coordinates (r, φ, z) in such a manner that the z axis is the axis of rotation of the cylinders. In this case, the equations governing the motion of the medium in the clearance have the form [4]

$$\begin{gathered}
\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{r\varphi}}{\partial \varphi} + \frac{\partial \sigma_{rz}}{\partial z} + \frac{\sigma_{rr} - \sigma_{\varphi\varphi}}{r} + \rho F_r = \rho a_r,\\
\frac{\partial \sigma_{\varphi r}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{\varphi\varphi}}{\partial \varphi} + \frac{\partial \sigma_{\varphi z}}{\partial z} + \frac{2}{r}\,\frac{\sigma_{\varphi r}}{r} + \rho F_\varphi = \rho a_\varphi,\\
\frac{\partial \sigma_{zr}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{z\varphi}}{\partial \varphi} + \frac{\sigma_{zr}}{r} + \rho F_z + \frac{\partial \sigma_{zz}}{\partial z} = \rho a_z,
\end{gathered} \tag{1}$$

[1]Institute of Mechanics of Metal–Polymer Systems, National Academy of Sciences of Belarusia, Gomel' 246050. [2]Institute of Mechanics, Moscow State University, Moscow 119899. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 43, No. 3, pp. 117–124, May–June, 2002. Original article submitted June 13, 2001; revision submitted January 22, 2002.

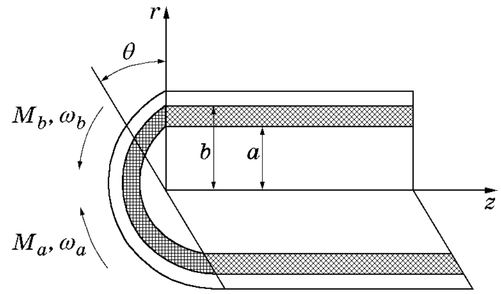

Fig. 1. Model of interaction between coaxial cylinders and a viscoplastic material in the clearance.

where σ_{ij} is the stress tensor, ρ is the density of the medium, F_i are the mass forces, and a_i are the components of the acceleration vector of the material particles.

We use the following relations for an incompressible material:

$$S_{ij} = \sigma_{ij} - \sigma\delta_{ij} = (2\tau/H)\xi_{ij}, \qquad \sigma = \sigma_{kk}/3,$$
$$\tau = \tau_0 + K(H/\dot{\varepsilon}_0)^m \quad \text{for } \tau \geqslant \tau_0, \qquad H = 0 \quad \text{for} \quad \tau < \tau_0. \tag{2}$$

Here ξ_{ij} are the components of the strain-rate tensor deviator, S_{ij} are the components of the stress tensor deviator, δ_{ij} is the Kronecker symbol, $\dot{\varepsilon}_0$ is the characteristic strain rate, τ_0 is the yield point (critical stress), τ is the shear-stress intensity, H is the intensity of the shear-strain rates, and m and K are the material constants.

Boundary Conditions. We assume that the kinematic boundary conditions of adhesion are satisfied. Thus, the boundary conditions for the velocity field U_r, U_φ, U_z have the form $U_\varphi(a) = \omega_a a$ and $U_\varphi(b) = \omega_b b$.

The aim of this study is to determine parameters of the mathematical model of a viscoplastic material τ_0, K, and m for the known parameters ω_a, ω_b, M_a, and M_b.

Method of the Solution. It follows from the formulation of the problem that $U_z = U_r = 0$ and $U_\varphi = U_\varphi(r)$. Therefore, the only nonzero component of the acceleration is $a_r = -U_\varphi^2/r$ and the only nonzero component of the strain-rate tensor is $\xi_{r\varphi} = (dU_\varphi/dr - U_\varphi/r)/2$. By virtue of (2), the stress tensor becomes

$$\sigma_{ij} = \begin{pmatrix} -p & \sigma_{r\varphi} & 0 \\ \sigma_{r\varphi} & -p & 0 \\ 0 & 0 & -p \end{pmatrix}. \tag{3}$$

With allowance for (1) and (3), we obtain the equations of motion of a continuum

$$\frac{dp}{dr} = -\rho\,\frac{U_\varphi^2}{r}, \qquad \frac{d\sigma_{r\varphi}}{dr} + 2\,\frac{\sigma_{r\varphi}}{r} = 0,$$

where p is the pressure. Integrating the last equation, we infer that, regardless of the determining relation of the material, the relation $\sigma_{r\varphi} = C_1/r^2$ holds, where C_1 is the integration constant, which generally depends on time.

We calculate the shear-stress intensity $\tau = \sqrt{S_{ij}S_{ij}/2} = |\sigma_{r\varphi}| = |C_1|/r^2$. The intensity of the shear-strain rates is given by $H = \sqrt{2\xi_{ij}\xi_{ij}} = r|d(U_\varphi/r)/dr|$.

Solution of the Direct Problem. We consider the solution of the direct problem (solutions of the direct problems for linear and nonlinear viscous liquids and Shvedov–Bingham plastic are given in [6]).

Taking (2) into account, from the expression for $\sigma_{r\varphi}$, we obtain

$$\tau_0 + Kr^m\,\frac{1}{\dot{\varepsilon}_0^m}\Big(\frac{d}{dr}\Big(\frac{U_\varphi}{r}\Big)\Big)^m = \frac{C_1}{r^2}$$

since $H = r|d(U_\varphi/r)/dr|$ or

$$r\,\frac{d}{dr}\Big(\frac{U_\varphi}{\dot{\varepsilon}_0 r}\Big) = \Big(\frac{1}{K}\Big)^n\Big(\frac{C_1}{r^2} - \tau_0\Big)^n, \tag{4}$$

where $n = 1/m$.

Integration of (4) yields

$$\frac{U_\varphi}{\dot{\varepsilon}_0} = \frac{r}{(K)^n} \int \frac{1}{r} \Big(\frac{C_1}{r^2} - \tau_0\Big)^n \, dr. \tag{5}$$

For numerical calculations, we choose the following parameters: $a = 10$ cm, $\tau = 100$ kg/cm^2, and $\dot{\varepsilon}_0 = 10^{-1}$ sec^{-1}. We introduce the dimensionless quantities $K' = (K/\tau) \cdot 10^3$, $\tau_0' = (\tau_0/\tau) \cdot 10^3$, $U_\varphi' = U_\varphi/(a\dot{\varepsilon}_0)$, $r' = r/a$, $H' = H/\dot{\varepsilon}_0$, and $C_1' = (C_1/(\tau a^2)) \cdot 10^3$.

In the dimensionless variables, equality (5) is written in the form

$$\frac{U_\varphi'}{r'} = \Big(\frac{C_1'}{K'}\Big)^n \int \frac{1}{(r')^{2n+1}} \Big(1 - \frac{\tau_0'(r')^2}{C_1'}\Big)^n \, dr'. \tag{6}$$

Substituting (6) into the boundary conditions

$$U_\varphi'(b/a)/r' = \omega_b/\dot{\varepsilon}_0, \tag{7}$$

$$U_\varphi'(1)/r' = \omega_a/\dot{\varepsilon}_0 \tag{8}$$

and subtracting (8) from (7), we obtain the equation

$$\Big(\frac{C_1'^{(1)}}{K'}\Big)^n \int\limits_1^{b/a} \frac{1}{(r')^{2n+1}} \Big(1 - \frac{\tau_0'(r')^2}{C_1'}\Big)^n \, dr' = \frac{\Delta\omega ba}{\dot{\varepsilon}_0}, \tag{9}$$

where $\Delta\omega ba = \omega_b - \omega_a$ and $C_1'^{(i)}$ is the value of the constant determined in the ith experiment.

Performing two additional numerical experiments, we obtain two similar equations

$$\Big(\frac{C_1'^{(2)}}{K'}\Big)^n \int\limits_1^{b/a} \frac{1}{(r')^{2n+1}} \Big(1 - \frac{\tau_0'(r')^n}{C_1'^{(2)}}\Big)^n \, dr' = \frac{\Delta\omega^{(2)} ba}{\dot{\varepsilon}_0}; \tag{10}$$

$$\Big(\frac{C_1'^{(3)}}{K'}\Big)^n \int\limits_1^{b/a} \frac{1}{(r')^{2n+1}} \Big(1 - \frac{\tau_0'(r')^2}{C_1'^{(3)}}\Big)^n \, dr' = \frac{\Delta\omega^{(3)} ba}{\dot{\varepsilon}_0}. \tag{11}$$

Eliminating K' by dividing (9) and (10) by (11), we arrive at two equations for two unknowns

$$\int\limits_1^{b/a} \frac{1}{(r')^{2n+1}} \Big(1 - \frac{\tau_0'(r')^2}{C_1^{'(1)}}\Big)^n dr' - \frac{\Delta\omega^{(1)} ba}{\Delta\omega^{(3)} ba} \Big(\frac{C_1^{'(3)}}{C_1^{'(1)}}\Big)^n \int\limits_1^{b/a} \frac{1}{(r')^{2n+1}} \Big(1 - \frac{\tau_0'(r')^2}{C_1^{'(3)}}\Big)^n dr' = 0; \tag{12}$$

$$\int\limits_1^{b/a} \frac{1}{(r')^{2n+1}} \Big(1 - \frac{\tau_0'(r')^2}{C_1^{'(2)}}\Big)^n dr' - \frac{\Delta\omega^{(2)} ba}{\Delta\omega^{(3)} ba} \Big(\frac{C_1^{'(3)}}{C_1^{'(2)}}\Big)^n \int\limits_1^{b/a} \frac{1}{(r')^{2n+1}} \Big(1 - \frac{\tau_0'(r')^2}{C_1^{'(3)}}\Big)^n dr' = 0, \tag{13}$$

where $C_1'^{(i)}$ are determined from the equations

$$\int\limits_1^{b/a} \frac{1}{(r')^{2n+1}} \Big(1 - \frac{\tau_0'(r')^2}{C_1'^{(i)}}\Big)^n dr' = \Big(\frac{K'}{C_1'^{(i)}}\Big)^n \frac{\Delta\omega^{(i)} ba}{\dot{\varepsilon}_0}, \qquad i = 1, 2, 3.$$

We reduce the last equation to the form

$$\frac{1}{C_1'^{(i)}} = \frac{1}{K'} \Bigg[\Big(\frac{\Delta\omega ba}{\dot{\varepsilon}_0}\Big)^{-1} \int\limits_1^{b/a} \frac{1}{(r')^{2n+1}} \Big(1 - \frac{\tau_0'(r')^2}{C_1'^{(i)}}\Big)^n dr'\Bigg]^{1/n}. \tag{14}$$

We solve these equations for the independent variable which enters the integral by the simple iterative method. Using the iterative formula $1/C_{1n+1}'^{(i)} = \varphi(C_{1n}'^{(i)})$, where $\varphi(C_{1n}'^{(i)})$ is the right side of Eq. (14), we find $C_1'^{(i)}$. Changing the variable $1/C_1'^{(i)} = x$, we obtain

$$x_{n+1} = \frac{1}{K'} \Bigg[\Big(\frac{\Delta\omega ba}{\dot{\varepsilon}_0}\Big)^{-1} \int\limits_1^{b/a} \frac{1}{(r')^{2n+1}} (1 - x_n \tau_0'(r')^2)^n \, dr'\Bigg]^{1/n}.$$

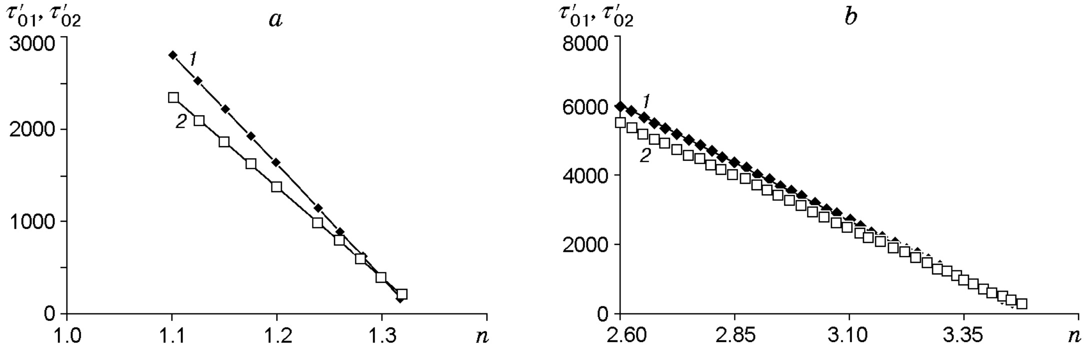

Fig. 2. The critical stresses τ'_{01} (curves 1) and τ'_{02} (curves 2) versus the parameter n in the neighborhood of the intersection point (τ'_0, n): (a) $\tau'_0 = 400$ and $n = 1.3$; (b) $\tau'_0 = 700$ and $n = 3.4$.

Introducing the new variables $K' = 10^{13/3}K_1$ and $x = x_1/10^{13/3}$, we obtain

$$x_{1,n+1} = \frac{1}{K_1}\Bigg[\Big(\frac{\Delta\omega ba}{\dot\varepsilon_0}\Big)^{-1}\int\limits_1^{b/a}\frac{1}{(r')^{2n+1}}\Big(1 - \frac{x_{1,n}\tau'_0(r')^2}{10^{13/3}}\Big)^n\,dr'\Bigg]^{1/n}.$$

The calculations show that, for $K_1 = 1$, $\Delta\omega ba/\dot\varepsilon_0 = 1$, 0.5, and 0.25, and $n = 1.1$–4.0 and $\tau'_0 = 100$–6000, the quantity x_1 is of order of unity. The values of x_1 were calculated with an accuracy of 10^{-9}. This accuracy is satisfied after 10 iterations for any n and small values of τ'_0 ($\tau'_0 = 100$–1000) and after about 20 iterations for large values of τ'_0.

The effect of an error in specifying the initial data on the convergence of the algorithm was studied. If the initial data deviate from their exact values by 10–30%, the number of iterations is equal to 6 or 7 for small values ($\tau'_0 = 100$–1000) and 20–25 for $\tau'_0 = 3000$–6000.

Solution of the Inverse Problem. Leonova [5] proposed a method for verification of the model of a linear-viscoplastic material. The problem considered in the present paper is more complex. To solve it, the following method is proposed. The values of the functions $F_1(\tau'_0, n)$ and $F_2(\tau'_0, n)$, which are the left sides of Eqs. (12) and (13), are calculated and the intervals where these functions change their sign are found. The curves of intersection of the plane $z = 0$ with the surfaces $z = F_1$ and $z = F_2$ are determined by the bisection method. The points of intersection of these curves are found. To solve this problem, the left sides of Eqs. (12) and (13) were calculated for different values of τ'_0 and n. For specific values of x_1, the functions $F_1(\tau'_0, n)$ and $F_2(\tau'_0, n)$ that enter (12) and (13) were calculated for the step in τ'_0 equal to 100. In particular, calculations were performed for five pairs of values: 1) $\tau'_0 = 400$ and $n = 1.3$; 2) $\tau'_0 = 800$ and $n = 1.6$; 3) $\tau'_0 = 1000$ and $n = 1.9$; 4) $\tau'_0 = 400$ and $n = 2.3$; 5) $\tau'_0 = 700$ and $n = 3.4$.

The regions in which the functions F_1 and F_2 change their sign were determined and the roots of the equations $F_1(\tau'_0, n) = 0$ and $F_2(\tau'_0, n) = 0$ were found by the bisection method. It was shown that each curve of intersection of the surface $z = F_1$ ($i = 1$, 2) with the plane $z = 0$ is close to a straight line and it is a single-valued function of $\tau'_0(n)$. The solution was determined with a high accuracy. In all the cases, for a fixed value of n, the functions $F_1(\tau'_0, n)$ and $F_2(\tau'_0, n)$ decrease as τ'_0 increases and increase with n.

To find common points of the functions $F_1(\tau'_0, n) = 0$ and $F_2(\tau'_0, n) = 0$, the graphs of these functions were plotted in the coordinates τ'_0 and n.

The behavior of the functions $\tau'_{01} = f_1(n)$ and $\tau'_{02} = f_2(n)$ in the neighborhood of the point of their intersection was also studied. It was found that the derivatives of the functions differ by approximately a factor of 1.5 at the points of intersection of the curves $\tau'_{01}(n)$ and $\tau'_{02}(n)$. It follows that these curves do not coincide with one another in the neighborhood of the intersection points. For $n = 1.1$–4.0 and $\tau'_0 = 100$–6000, no other points of intersection of the curves $F_1(\tau'_0, n) = 0$ and $F_2(\tau'_0, n) = 0$ were found.

Figure 2 shows the curves $\tau'_{01}(n)$ and $\tau'_{02}(n)$ [$F_1(\tau'_0, n) = 0$ and $F_2(\tau'_0, n) = 0$] in the neighborhood of their intersection points.

We consider the approximate solution of the equation

$$\frac{U'_\varphi}{r'} = \Big(\frac{C'_1}{K'}\Big)^n\int\frac{1}{(r')^{2n+1}}\Big(1 - \frac{\tau'_0(r')^2}{C'_1}\Big)^n\,dr'. \qquad (15)$$

It should be noted that the integral in (15) cannot be evaluated analytically for an arbitrary n. Taking into account that $\tau_0'(r')^2/C_1' < 1$, performing expansion using the binomial theorem, and retaining two terms in the series, we obtain the approximate expression for $U_\varphi'(r')$:

$$\frac{U_\varphi'}{r'} = \Big(\frac{C_1'}{K'}\Big)^n\Big[\int \frac{dr'}{(r')^{2n+1}} - \frac{n\tau_0'}{C_1'}\int \frac{dr'}{(r')^{2n-1}} + \frac{n(n-1)}{2}\Big(\frac{\tau_0'}{C_1'}\Big)^2\int \frac{dr'}{(r')^{2n-3}}\Big]. \tag{16}$$

Integrating and multiplying both sides of (16) by r', we obtain

$$U_\varphi'(r') = \Big(\frac{C_1'}{K'}\Big)^n\Big[-\frac{1}{2n}\,\frac{1}{(r')^{2n-1}} + \frac{n\tau_0'}{C_1'}\,\frac{1}{2n-2}\,\frac{1}{(r')^{2n-3}} - \frac{n(n-1)}{2(2n-4)}\Big(\frac{\tau_0'}{C_1'}\Big)^2\frac{1}{(r')^{2n-5}} + C_2'r'\Big].$$

The constants C_1' and C_2' are found from the kinematic boundary conditions $U_\varphi(a) = \omega_a a$ and $U_\varphi(b) = \omega_b b$ or $U_\varphi'(1) = \omega_a/\dot\varepsilon_0$ and $U_\varphi(b/a) = \omega_b b/(a\dot\varepsilon_0)$. The boundary condition implies

$$U_\varphi'(1) = \Big(\frac{C_2'}{K'}\Big)^n\Big[-\frac{1}{2n} + \frac{n\tau_0'}{C_1'}\,\frac{1}{2n-2} - \frac{n(n-1)}{2(2n-4)}\Big(\frac{\tau_0'}{C_1'}\Big)^2 + C_2'\Big] = \frac{\omega_a}{\dot\varepsilon_0}; \tag{16a}$$

$$U_\varphi'\Big(\frac{b}{a}\Big) = \Big(\frac{C_1'}{K}\Big)^n\Big[-\frac{1}{2n}\Big(\frac{a}{b}\Big)^{2n-1} + \frac{n\tau_0'}{2n-2}\,\frac{1}{C_1'}\Big(\frac{a}{b}\Big)^{2n-3} - \frac{n(n-1)}{2(2n-4)}\Big(\frac{\tau_0'}{C_1'}\Big)^2\Big(\frac{a}{b}\Big)^{2n-5} + C_2'\,\frac{b}{a}\Big) = \frac{\omega_b}{\dot\varepsilon_0}\,\frac{b}{a}. \tag{16b}$$

Dividing (16b) by b/a and subtracting (16a) from (16b), we obtain the equation for C_1':

$$\Big(\frac{C_1'}{K'}\Big)^n\Big[\frac{1}{2n}\Big(1-\Big(\frac{a}{b}\Big)^{2n}\Big) + \frac{n\tau_0'}{2n-2}\,\frac{1}{C_2'}\Big(\Big(\frac{a}{b}\Big)^{2n-2} - 1\Big) + \frac{n(n-1)}{2(2n-4)}\Big(\frac{\tau_0'}{C_1'}\Big)^2\Big(1-\Big(\frac{a}{b}\Big)^{2n-4}\Big)\Big] = \frac{\omega_b' - \omega_a'}{\dot\varepsilon_0} = \frac{\Delta\omega' ba}{\dot\varepsilon_0}. \tag{17}$$

We introduce the parameters $\tilde x = 1-(a/b)^{2n}$, $\tilde y = (a/b)^{2n-2} - 1$, and $\tilde z = 1-(a/b)^{2n-4}$. Then, Eq. (17) becomes

$$\frac{(C_1')^n}{2n}\,\tilde x + \frac{n\tau_0'}{2n-2}\,\tilde y (C_1')^{n-1} + \frac{n(n-1)}{2(2n-4)}\,(\tau_0')^2(C_1')^{n-2}\tilde z = (K')^n\Delta\omega' ba.$$

Since the data of one experiment determine only one equation for n, τ_0', and K', to determine the parameters of the model it is necessary to perform three experiments for different angular velocities of coaxial cylinders. As a result, we obtain the system of three equations

$$\begin{gathered}
\frac{(C_1'^{(1)})^n}{2n}\,\tilde x + \frac{n\tau_0'}{2n-2}\,\tilde y (C_1'^{(1)})^{n-1} + \frac{n(n-1)}{2(2n-4)}\,(\tau_0')^2(C_1'^{(1)})^{n-2}\tilde z = (K')^n\Delta\omega^{(1)} ba,\\
\frac{(C_1'^{(2)})^n}{2n}\,\tilde x + \frac{n\tau_0'}{2n-2}\,\tilde y (C_1'^{(2)})^{n-1} + \frac{n(n-1)}{2(2n-4)}\,(\tau_0')^2(C_1'^{(2)})^{n-2}\tilde z = (K')^n\Delta\omega^{(2)} ba,\\
\frac{(C_1'^{(3)})^n}{2n}\,\tilde x + \frac{n\tau_0'}{2n-2}\,\tilde y (C_1'^{(3)})^{n-1} + \frac{n(n-1)}{2(2n-4)}\,(\tau_0')^2(C_1'^{(3)})^{n-2}\tilde z = (K')^n\Delta\omega^{(3)} ba.
\end{gathered} \tag{18}$$

Dividing the first equation in (18) by the second and third equations, we obtain two equations for n_1 and τ_0'. For $\Delta\omega'^{(1)}ba/(\Delta\omega'^{(2)}ba) = 2$ and $\Delta\omega'^{(1)}ba/(\Delta\omega'^{(3)}ba) = 4$ and $C_1'^{(i)} = 10^{13/3}x_i$, these equations become

$$\begin{aligned}
&10^{-13/3}\,\frac{\tilde x}{2n}((x_1)^n - 2(x_2)^n) + \frac{n\tau_0'\tilde y}{2n-2}\,((x_1)^{n-1} - 2(x_2)^{n-1})\\
&\quad + 10^{-26/3}\,\frac{n(n-1)}{2(2n-4)}(\tau_0')^2\tilde z((x_1)^{n-2} - 2(x_2)^{n-2}) = 0;
\end{aligned} \tag{19}$$

$$\begin{aligned}
&10^{-13/3}\,\frac{\tilde x}{2n}((x_1)^n - 4(x_3)^n) + \frac{n\tau_0'\tilde y}{2n-2}\,((x_1)^{n-1} - 4(x_3)^{n-1})\\
&\quad + 10^{-26/3}\,\frac{n(n-1)}{2(2n-4)}(\tau_0')^2\tilde z((x_1)^{n-2} - 4(x_3)^{n-2}) = 0.
\end{aligned} \tag{20}$$

Expressing τ_0' from Eq. (19) in terms of n and substituting it into Eq. (20), we obtain an equation for one variable n. Using the iteration scheme $n_{m+1} = n_m + F(n_m, \tau_0'(n_m))$, one can find the root n.

The stability of the algorithm was studied. In the interval $n = 1.05$–4, arithmetical means equal to the deviations of n_1, τ_0', and K' from the exact values were calculated with a step of 0.05. For this purpose, 100 experiments with a random deviation of 1, 2, and 3% in x_i were performed.

The calculations show that, in the entire range of n, the accuracy in determining τ_0' is unsatisfactory for small values of τ_0' (the error exceeds 1000%), but this has little effect on the calculation of n and K'. The values of τ_0' comparable to K are determined with a better accuracy. The possible reason is that, in the equation for τ_0', the terms containing τ_0' are small compared to the first term, which is independent of this parameter.

If the moments deviate from the exact values by less than 1%, the measurement error does not exceed 10%. For deviations of 2 and 3%, the error is smaller than 20 and 30%, respectively. The algorithm proposed can be used for $\tau_0' = 11{,}000$–$22{,}000$ provided the other constants n and K' correspond to superplastic materials.

Verification of the Method Using Experimental Data. To verify the method proposed, experiments were performed using the rotational viscometer Rheotest 2.1, in which the Couette flow was obtained. As a viscoplastic material, we used a grease based on the I-40A industrial oil with surfactants and stiffeners. Experiments were performed at $t = 25$ °C in the range of shear stresses of 30–170 Pa. The ratio of the inner and outer radii of the coaxial cylinders was 0.98.

The following angular velocities were specified in the experiments: $\Delta\omega_{ba}^1 = -25.434$ sec^{-1}, $\Delta\omega_{ba}^2 = -8.378$ sec^{-1}, and $\Delta\omega_{ba}^3 = -4.71$ sec^{-1}. The following shear stresses corresponding to these velocities were recorded: $\sigma_{r\varphi}^1|_{r=a} = c_1/a^2 = 169.4$ Pa, $\sigma_{r\varphi}^2|_{r=a} = 72.6$ Pa, and $\sigma_{r\varphi}^3|_{r=a} = 49.7$ Pa. Using the above-proposed method based on solving a system of nonlinear algebraic equations, we found the dimensionless material constants n, τ_0, and K'.

For calculations, we used the characteristic values of the parameters $\tau = 10$ Pa, $\dot{\varepsilon} = 1$ sec^{-1}, and $a = 1$ cm which correspond to the dimensionless parameters $\tau_0' = 100\tau_0$, $c_1' = 100c_1$, and $K' = 100K$. The following material constants were obtained: $\tau_0' = 79.3$, $n = 3.73$, and $K' = 19.51$. Since τ_0 is of the same order as the parameter K, the solution is expected to be stable, which is supported by the calculations considered above.

It was found that a 1% error in measuring the shear stresses leads to errors of 3, 2–6, and 3–10% in determining the parameters n, τ_0', and K', respectively. Since the material constants were unknown, a fourth experiment was performed to solve the direct problem and compare the calculated shear stress with its experimental value. For example, the shear stress was calculated to be $\sigma_{r\varphi}|_{r=a} = 36.3$ Pa for $\Delta\omega_{ba}^4 = -2.926$ sec^{-1}. In solving the direct problem, the shear stress was determined with a relative error of 5%, which shows that the method developed provides a high accuracy.

The constants obtained by solving the exact equations for the same angular velocities and shear stresses differ from the approximate values by 10% or smaller. Consequently, the approximate method can be used to estimate the characteristics of a viscoplastic material.

Conclusions. An algorithm of identifying the model of a viscoplastic material from experimental data on the Couette flow is proposed. It was found that the algorithm of determining the moment of rotation of coaxial cylinders $C_1^{(i)}$ is stable and converges in the range of τ_0' studied. To find the solution with an accuracy of up to 10^{-9}, 25 iterations are required. Equations for finding τ_0' and n are such that a single root exists in the range of τ_0' and n considered, whereas each curve of intersection of the function $z_i = F_i(x, y)$ with the plane $z_i = 0$ is close to a straight line.

Experimental verification of the method proposed by studying a viscoplastic grease under the conditions of the Couette flow using a rotational viscometer shows that the material constants are determined with an accuracy sufficient for practical purposes.

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